Macroeconomics 1 (3/7)

The DICE model (Nordhaus)

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September – December 2024

Growth and climate change

- The CKR model does not take into account the consequences of economic activity for the climate nor, vice-versa, the consequences of **climate change** for the economy.
- Nordhaus (1992, 1994) has extended the CKR model to take these consequences into account, giving rise to the **DICE model** ($\equiv$ Dynamic Integrated Climate-Economy model), which is a model of the world economy and the world climate.
- **William D. Nordhaus**: American economist, born in 1941 in Albuquerque, professor at Yale University since 1967, co-laureate (with Paul M. Romer) of the Sveriges Riksbank's prize in economic sciences in memory of Alfred Nobel in 2018 "*for integrating climate change into long-run macroeconomic analysis*".

Pollution externality

- A key difference with the CKR model is the presence, in the DICE model, of a **pollution externality**.
- The production activity of each firm, by emitting greenhouse gases, contributes to climate change which harms all agents.
- Because of this externality,
  - the first welfare theorem does not apply,
  - the competitive equilibrium under laisser-faire is not socially optimal,
  - the $\mathcal{BOOP}$ would choose less production and less greenhouse-gas emissions,
  - the optimal "**carbon tax**" is positive.

Overview of the chapter

- This chapter presents
  - the **equilibrium conditions** of the DICE model,
  - its **normative implications** (optimal carbon tax).

- The optimal value of the carbon tax in the DICE model is very sensitive to the value chosen for the **discount rate**.

- For this reason, the chapter also discusses how to calibrate the discount rate
  - depending on the (descriptive or prescriptive) approach considered,
  - taking or not taking into account uncertainty.

Which DICE model?

- Nordhaus has, over time, developed several successive versions of the DICE model:
  - the first one, DICE 1992 (Nordhaus, 1992, 1994), is the simplest,
  - the last two, DICE 2016 and DICE 2023, are the most complicated.

- In the following, we present
  - the equilib. conditions of DICE 1992 (reformulated in continuous time),
  - the calibration and results of DICE 1992, DICE 2016 and DICE 2023.

Chapter outline

Equilibrium conditions

Economic part I

- The DICE model has two parts, which interact with each other:
  - an economic part,
  - a climatic part.

- The **economic part** of the DICE-1992 model corresponds to the CKR model with two simplifications and one change.

- **Simplifications:**
  - logarithmic consumption utility: $u(c_t) = \ln(c_t)$ (i.e. coefficient of relative risk aversion constant, equal to $\theta = 1$),
  - Cobb-Douglas production function for each firm i: $Y_{i,t} = \Omega_t K_{i,t}^{\alpha}(A_t N_{i,t})^{1-\alpha}$, with $0 < \alpha < 1$.

Economic part II

- **Change:** in the production function, instead of $\Omega_t \equiv 1$, we have

$$\Omega_t \equiv \frac{1 - b_1 \mu_t^{b_2}}{1 + \theta_1 T_t^{\theta_2}}$$

with $b_1 > 0$, $b_2 > 0$, $\theta_1 > 0$, $\theta_2 > 0$, where
  - T_t is the temperature of the surface and shallow oceans,
  - μ_t the greenhouse-gas-emission reduction rate.

- **Interpretation:**
  - $\partial \Omega_t / \partial T_t < 0$ captures the economic cost of climate change,
  - $\partial \Omega_t / \partial \mu_t < 0$ captures the economic cost of greenhouse-gas-emission reduction.

- In this model, the **emission reduction rate** μ_t is considered as the economic-policy instrument; it can be interpreted as the outcome of an emission tax ("carbon tax").

Economic part III

- Each firm i being atomistic, its individual decisions have, everything else equal, a negligible effect on the temperature T_t and on the economic-policy instrument μ_t.

- Each firm i thus chooses $K_{i,t}$ and $N_{i,t}$ to maximize its instantaneous profit taking T_t and μ_t, and therefore Ω_t, as given.

- The first-order conditions of firms' optimization problem are thus the same as in Chapter 2, now with the new Ω_t factor.

- The other equilibrium conditions of Chapter 2, characterizing households' behavior and markets' clearing, are unchanged.

General overview of the economic part I *

- **Firms** rent capital and employ labor to produce goods, with a total factor productivity that depends negatively on
  - the temperature of the surface and shallow oceans,
  - greenhouse-gas-emission reduction rate.

- **Households** own capital and supply labor.

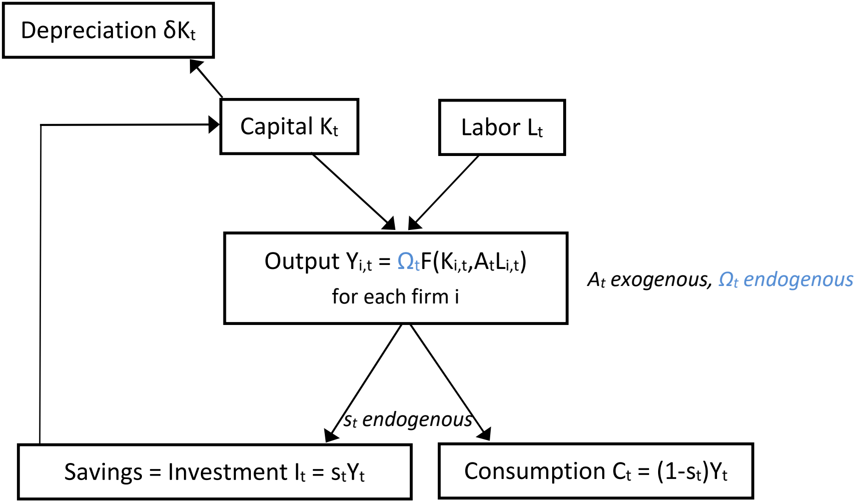
- The goods produced by firms are used for households' consumption and investment in new capital.

- The **saving rate** is **endogenous**, optimally chosen by **households**.

- Capital evolves over time due to investment and capital depreciation.

(In the pages whose title is followed by an asterisk,
in blue: changes from Chapter 2.)

General overview of the economic part II *

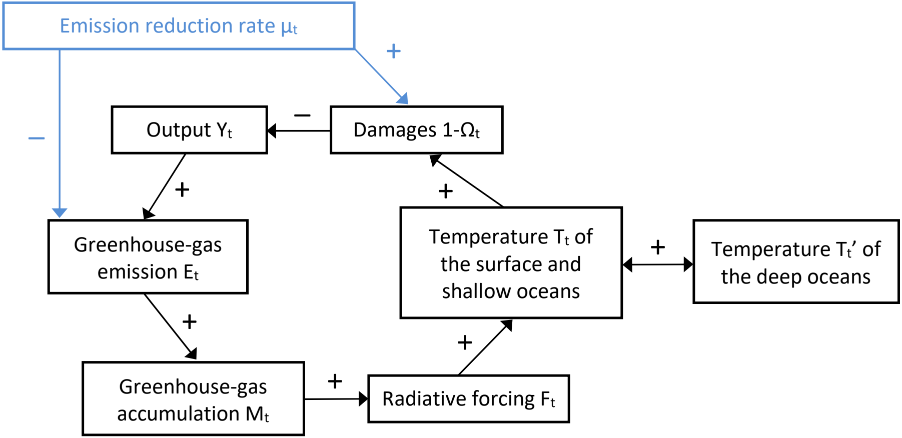


General overview of the climatic part I

- Production (flow Y_t) emits greenhouse gases (flow E_t), all the more so as the emission reduction rate μ_t is low.
- These gases accumulate in the atmosphere (stock M_t).
- This accumulation increases radiative forcing F_t.
- This increase in radiative forcing raises
  - the temperature T_t of the surface and shallow oceans,
  - the temperature T'_t of the deep oceans,

  which are linked to each other.
- The rise in T_t leads, everything else equal, to a decrease in output Y_t.

General overview of the climatic part II



(In blue: economic-policy instrument.)

Equations of the climatic part I

- **Emissions** of greenhouse gases:

$$E_t = (1 - \mu_t)\varphi_t Y_t,$$

where φ_t is exogenous.

- **Accumulation** of greenhouse gases in the atmosphere:

$$\dot{M}_t = \gamma E_t - \delta_m (M_t - M)$$

with $\gamma > 0$ and $\delta_m > 0$, where M represents the pre-industrial value of M_t.

- **Radiative forcing**:

$$F_t = \eta \log_2 \frac{M_t}{M} + O_t$$

with $\eta > 0$, where O_t is exogenous.

Equations of the climatic part II

- Dynamics of the **temperature T_t of the surface and shallow oceans**:

$$\dot{T}_t = \frac{1}{R_1}\left[F_t - \lambda T_t - \frac{R_2}{\tau}\left(T_t - T'_t\right)\right]$$

with $R_1 > 0$, $R_2 > 0$, $\lambda > 0$, and $\tau > 0$.

- Dynamics of the **temperature T'_t of the deep oceans**:

$$\dot{T}'_t = \frac{1}{\tau}\left(T_t - T'_t\right).$$

Normative implications

Pollution externality

- For some given $(K_{j,t}, N_{j,t})_{j \neq i}$, a variation in $(K_{i,t}, N_{i,t})$ has both
  - a direct effect on $Y_{i,t} = \Omega_t K_{i,t}^{\alpha} (A_t N_{i,t})^{1-\alpha}$,
  - an indirect effect on all the $Y_{j,t'}$ for $j \in \{1, ..., I\}$ and $t' \geq t$, via Y_t, E_t, $(M_{t'})_{t' \geq t}$, $(F_{t'})_{t' \geq t}$, $(T_{t'})_{t' \geq t}$ and $(\Omega_{t'})_{t' \geq t}$.
- Firm i takes only the first effect into account when choosing $(K_{i,t}, N_{i,t})$ because
  - it does not take into account the indirect effect on the $Y_{j,t'}$ for $j \neq i$,
  - the indirect effect of $(K_{i,t}, N_{i,t})$ on $Y_{i,t'}$ is negligible compared with the direct effect of $(K_{i,t'}, N_{i,t'})$ on $Y_{i,t'}$ (the number of firms I being large).
- As a consequence, each firm i chooses $K_{i,t}$ and $N_{i,t}$ to maximize its instantaneous profit taking Ω_t as given.
- We say that there is a **pollution externality** between firms.

Implications for the optimal carbon tax

- Because of this externality,
    - the first welfare theorem does not apply,
    - the comp. equilibrium with $\mu_t = 0$ for $t \geq 0$ is not socially optimal,
    - the optimal (i.e. U_0-maximizing) path $(\mu_t)_{t \geq 0}$ is non-zero.

- The numerical results for the optimal carbon tax depend on
    - the model version,
    - the calibration of this version.

- They particularly depend on the calibration of
    - the damages caused by climate change (parameters θ_1 and θ_2),
    - the discount rate ("parameter" r).

Calibration of DICE 1992, 2016 and 2023

	DICE 1992	DICE 2016	DICE 2023
Damages caused by a 3°C warming *(in % of production)*	1.3%	2.1%	3.1%
Discount rate *(in % per year)*			
average from 2020 to 2050	not avail.	4.7%	4.4%
average from 2020 to 2100	not avail.	4.2%	3.9%

Sources: Barrage and Nordhaus (2023), Nordhaus (1994, 2018, 2019).

Results of DICE 1992, 2016 and 2023

	DICE 1992	**DICE 2016**	**DICE 2023**
Optimal carbon **tax** *(in 2018 \$ per ton of CO_2)*			
in 2020	18\$	43\$	53\$
in 2050	32\$	105\$	127\$
in 2100	40\$	295\$	not avail.
Warming from the pre-industrial period to 2100 *(in ℃)*			
with the current tax	3.3℃	4.1℃	3.8℃
with the optimal tax	3.2℃	3.5℃	2.7℃

Sources: Barrage and Nordhaus (2023), Nordhaus (1994, 2018, 2019).

Sensitivity of the results to the calibration I

- Nordhaus has, over time, revised upwards his calibration of damages caused by climate change (as shown on page 20).
- Nonetheless, this calibration has been criticized for being too low.
- In the next two pages, we consider a higher calibration, inspired by Howard and Sterner (2017).
- This calibration sets the damages at 9% of production for a 3°C warming (instead of 3.1% in DICE 2023).
- In these two pages, we also consider alternative calibrations for the discount rate, ranging from 5% to 1% per year.

Sensitivity of the results to the calibration II

Optimal carbon tax *(in 2019 $ per ton of CO_2)*
depending on the calibration of DICE 2023

Calibration...	**2020**	**2025**	**2050**
...serving as benchmark	53	62	127
...with higher damages	132	156	293
...with alternative discount rates			
$r = 5\%$ per year	33	39	77
$r = 4\%$ per year	51	60	110
$r = 3\%$ per year	87	103	170
$r = 2\%$ per year	170	200	289
$r = 1\%$ per year	429	505	609

Source: Barrage and Nordhaus (2023).

Sensitivity of the results to the calibration III

Warming from the pre-industrial period *(in °C)*
under optimal tax, depending on the calibration of DICE 2023

Calibration...	**2020**	**2050**	**2100**	**2150**
...serving as a benchmark	1.2	1.9	2.7	2.8
...with higher damages	1.2	1.8	1.9	1.7
...with alternative discount rates				
$r = 5\%$ per year	1.2	2.0	3.0	3.6
$r = 4\%$ per year	1.2	2.0	2.9	3.3
$r = 3\%$ per year	1.2	1.9	2.6	2.7
$r = 2\%$ per year	1.2	1.9	2.2	2.0
$r = 1\%$ per year	1.2	1.9	1.8	1.6

Source: Barrage and Nordhaus (2023).

Discount rate

Role of the discount rate

- The numerical normative implications of the DICE model are very sensitive to the calibration of the **discount rate** (or real interest rate r_t).

- For a given value D_t of damages occurring at time $t > 0$ (caused by climate change), the lower $(r_\tau)_{0 \leq \tau \leq t}$,
  - the higher the actualized value $D_t e^{-\int_0^t r_\tau d\tau}$ of these future damages,
  - the higher the optimal tax path $(\mu_t)_{0 \leq \tau \leq t}$,
  - the lower the "optimal" temperature path $(T_t)_{0 \leq \tau \leq t}$.

Steady-state discount rate I

- With a CRRA instantaneous-utility function, the Euler equation is

$$\frac{\dot{c}_t}{c_t} = \frac{r_t - \rho}{\theta}.$$

- We admit that the DICE model has a steady state in which per-capita consumption c_t grows at the rate of technological progress g, like the CKR model (Chapter 2).

- At this steady state, the discount rate (i.e. the value of r_t) is therefore

$$r = \underbrace{\rho}_{\text{impatience effect}} + \underbrace{\theta g}_{\text{wealth effect}}.$$

Steady-state discount rate II

- The discount rate r depends positively on
  - the **rate of time preference** ρ: the more impatient the agents, ...
  - the **growth rate of the economy** g: the more agents will consume in the future relatively to the present, the lower the marginal utility of consumption in the future relatively to the present, ...
  - the **inverse of the elasticity of intertemporal substitution** θ: the higher θ, the more the marginal utility of consumption $(c_t^{-\theta})$ is decreasing in consumption (c_t), the lower the marginal utility of consumption in the future relatively to the present (for $g > 0$), ...

...the more preferable present consumption relatively to future consumption.

Examples of calibration of r

	ρ (% per year)	g (% per year)	θ	**discount rate** (% per year)
Weitzman (2007)	2%	2%	2	**6%**
Nordhaus (2007)	1.5%	2%	2	**5.5%**
Nordhaus (2008)	1%	2%	2	**5%**
Gollier (2013)	0%	2%	2	**4%**
Stern (2007)	0.1%	1.3%	1	**1.4%**

$\hookrightarrow$ Stern (2007) recommends a substantially higher carbon tax than Nordhaus (2007) because he considers a substantially lower discount rate.

Calibration of ρ I

- **Descriptive approach**: Nordhaus (2007) calibrates ρ using macroeconomic and financial data (real interest rate).

- **Prescriptive approach**: Stern (2007) considers that ρ represents
  - the weight of present generations' utility relatively to future generations' utility (in the social utility function),
  - and not the weight of present utility relatively to future utility for a given generation (in the individual utility function)

  (we will come back to this distinction in the overlapping-generations model in Chapter 7).

- The prescriptive approach suggests the calibration $\rho = 0$: there is no reason to put a lower weight on future generations' utility than on present generations' utility (in the social utility function).

Calibration of ρ II

- The calibration of ρ, however, must satisfy the constraint

$$\rho - n > (1 - \theta) g,$$

for households' intertemporal utility to take a finite value at the steady state (as seen in Chapter 2).

- For $\theta = 1$ (value chosen by Stern, 2007) and $n = 0$ (value chosen by Stern, 2007, for the post-2200 period), this constraint amounts to $\rho > 0$.

- Stern (2007) chooses the value $\rho = 0.1\%$ per year, which he justifies with a(n exogenous) risk of human extinction of 0.1% per year.

Taking uncertainty into account I

- The expression $r = \rho + \theta g$ was obtained by ignoring uncertainty; now, the future is obviously uncertain, all the more so with climate change.

- In the presence of **uncertainty**, we consider the following intertemporal utility ("**expected-utility theory**" of Morgenstern and Von Neumann, 1953):

$$U_0 \equiv \mathbb{E}_0 \left\{ \int_0^{+\infty} e^{-\rho t} u(c_t) dt \right\},$$

  where $\mathbb{E}_0\{.\}$ represents the **expectation** operator conditional on the information set at time 0.

- For the sake of simplicity, we have set $n = 0$ (which does not affect the results).

- Let us assume that the real interest rate is constant, and let r denote its value.

Taking uncertainty into account II

- The household has the possibility of deviating from their optimal choice (c_0, c_1) by
  - lending an additional infinitesimal quantity of goods ds at time 0,
  - consuming the additional infinitesimal quantity of goods $e^r ds$ at time 1.

- The change in intertemporal utility ΔU_0 that this deviation would entail is

$$\Delta U_0 = -u'(c_0)ds + e^{-\rho}\mathbb{E}_0\{u'(c_1)\}e^r ds = \left[-u'(c_0) + e^{r-\rho}\mathbb{E}_0\{u'(c_1)\}\right] ds.$$

- Since (c_0, c_1) is the household's optimal choice, we have $\Delta U_0 = 0$:
  - if $\Delta U_0 > 0$, then the hous. would prefer to deviate as described above,
  - if $\Delta U_0 < 0$, then the household would prefer to deviate in the opposite direction (borrow more at time 0 and consume less at time 1).

- We thus obtain the following **Euler equation** from time 0 to time 1:

$$u'(c_0) = e^{r-\rho}\mathbb{E}_0\{u'(c_1)\}.$$

Taking uncertainty into account III

- In the following particular case:
  - **no uncertainty**: $\mathbb{E}_0\{u'(c_1)\} = u'(c_1)$,
  - CRRA instantaneous-utility function: $u'(c_t) = c_t^{-\theta}$,
  - constant growth rate of per-capita consumption: $c_1 = e^g c_0$,

  this Euler equation can be rewritten as $c_0^{-\theta} = e^{r-\rho} c_0^{-\theta} e^{-\theta g}$, that is to say

$$r = \rho + \theta g.$$

- If u' is strictly convex, then, everything else equal, the larger the uncertainty about c_1 (i.e. the variance of c_1),
  - the larger $\mathbb{E}_0\{u'(c_1)\}$ (as a consequence of a generalized version of Jensen's inequality),
  - the smaller r (as a consequence of the Euler equation),
  - the more households want to save (**precautionary savings**).

Taking uncertainty into account IV

- The function u' being positive and strictly decreasing, it is strictly convex at least locally.

- In the CRRA case $(u'(c_t) = c_t^{-\theta})$, u' is strictly convex globally: $u'''(c_t) = \theta(\theta+1)c_t^{-\theta-2} > 0$ for any $c_t > 0$.

- A measure of the convexity of u' is the **coefficient of relative prudence** (Kimball, 1990):

$$p(c_t) \equiv \frac{-c_t u'''(c_t)}{u''(c_t)}.$$

- In the CRRA case, $p(c_t)$ is independent of c_t and equal to

$$p(c_t) = \theta + 1.$$

Taking uncertainty into account V

- We henceforth consider the following particular case:
  - CRRA instantaneous-utility function: $u'(c_t) = c_t^{-\theta}$,
  - growth rate of per-capita consumption from time 0 to time 1 following a normal distribution:

$$c_1 = e^{\tilde{g}} c_0 \quad \text{with} \quad \tilde{g} \sim \mathcal{N}\left(\mu, \sigma^2\right),$$

  where $\mu \in \mathbb{R}$ and $\sigma \in \mathbb{R}_+ \setminus \{0\}$.

- The Euler equation can then be rewritten as $c_0^{-\theta} = e^{r-\rho} c_0^{-\theta} \mathbb{E}\{e^{-\theta \tilde{g}}\}$, that is to say

$$r = \rho - \ln \mathbb{E}\{e^{-\theta \tilde{g}}\} = \rho + \theta \left(\mu - \frac{\theta}{2}\sigma^2\right),$$

where the last equality comes from the result $\mathbb{E}\{e^{-\theta \tilde{g}}\} = e^{-\theta\left(\mu - \frac{\theta}{2}\sigma^2\right)}$ proved in the appendix.

Taking uncertainty into account VI

- Let g denote the growth rate of *expected* per-capita consumption from time 0 to time 1: $\mathbb{E}\{c_1\} = e^g c_0$ and hence

$$g = \ln \frac{\mathbb{E}_0\{c_1\}}{c_0} = \ln \frac{\mathbb{E}_0\{e^{\tilde{g}} c_0\}}{c_0} = \ln \mathbb{E}\{e^{\tilde{g}}\} = \mu + \frac{1}{2}\sigma^2,$$

where the last equality comes from the result $\mathbb{E}\{e^{\tilde{g}}\} = e^{\mu + \frac{1}{2}\sigma^2}$ proved in the appendix.

- We can thus rewrite the Euler equation as

$$r = \underbrace{\rho}_{\text{impatience effect}} + \underbrace{\theta g}_{\text{wealth effect}} - \underbrace{\frac{1}{2}\theta\left(\theta + 1\right)\sigma^2}_{\text{precaution effect}}.$$

Taking uncertainty into account VII

- The **precaution effect** is equal to half the product of
  - the coefficient of relative risk aversion (θ),
  - the coefficient of relative prudence ($\theta + 1$),
  - the variance of the growth rate of the economy (σ^2).

- The same result is obtained, this time as a second-order approximation, when the CRRA-utility and normal-distribution assumptions are relaxed.

- Considering $\sigma = 3.6\%$ (standard error of the year-on-year growth rate of per-capita consumption in the US), Gollier (2013) gets a precaution effect of 0.4% per year and hence a discount rate of 3.6% per year.

- Gollier (2013) shows that the precaution effect can be larger, and hence the discount rate smaller, in the **long term** and/or in the presence of **catastrophic risks**.

Conclusion

Main predictions of the model

- The competitive equilibrium under laisser-faire is not socially optimal because of a **pollution externality**.

- Everything else equal, the **optimal carbon tax** depends
  - positively on the economic damages caused by climate change,
  - negatively on the **discount rate**.

- Under **certainty**, the discount rate (r) is the sum of
  - an **impatience effect** (ρ),
  - a **wealth effect** (θg).

- **Uncertainty** (normal distribution for the growth rate) reduces the discount rate (r) in the short term by a **precaution effect** ($\theta(\theta + 1)\sigma^2/2$).

One limitation of the model

- As in the CKR model (Chapter 2), **the rate of technological progress g is exogenous.**
- Now, this rate of technological progress is a key determinant of the discount rate and hence of the optimal carbon tax in the DICE model.
- If the rate of technological progress were endogenous,
    - could some policies affect it?
    - what role should they play?

↪ Chapters 4 and 5 ("**endogenous-growth theories**") endogenize the rate of technological progress.

Appendix

Computation of $\mathbb{E}\{e^{-\varphi\tilde{g}}\}$ when $\tilde{g} \sim \mathcal{N}(\mu, \sigma^2)$ I

- For any $\mu^* \in \mathbb{R}$ and any $\sigma^* \in \mathbb{R}_+ \setminus \{0\}$, let

$$x \mapsto f(x; \mu^*, \sigma^*) \equiv \frac{1}{\sigma^* \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu^*}{\sigma^*}\right)^2}$$

denote the density of the distribution $\mathcal{N}(\mu^*, \sigma^{*2})$.

- For any $\mu^* \in \mathbb{R}$ and any $\sigma^* \in \mathbb{R}_+ \setminus \{0\}$, since $f(x; \mu^*, \sigma^*)$ is a density, we have

$$\int_{-\infty}^{+\infty} f(x; \mu^*, \sigma^*) dx = 1.$$

Computation of $\mathbb{E}\{e^{-\varphi\tilde{g}}\}$ when $\tilde{g} \sim \mathcal{N}(\mu,\sigma^2)$ II

- If $\tilde{g} \sim \mathcal{N}(\mu,\sigma^2)$, then for any $\varphi \in \mathbb{R}$,

$$\begin{aligned}
\mathbb{E}\{e^{-\varphi\tilde{g}}\} &= \int_{-\infty}^{+\infty} e^{-\varphi x} f(x;\mu,\sigma)\, dx \\
&= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\varphi x} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx \\
&= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\varphi\left(\mu-\frac{\varphi}{2}\sigma^2\right)} e^{-\frac{1}{2}\left[\frac{x-\left(\mu-\varphi\sigma^2\right)}{\sigma}\right]^2} dx \\
&= e^{-\varphi\left(\mu-\frac{\varphi}{2}\sigma^2\right)} \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} f\left(x;\mu-\varphi\sigma^2,\sigma\right) dx \\
&= e^{-\varphi\left(\mu-\frac{\varphi}{2}\sigma^2\right)}.
\end{aligned}$$

- Replacing φ with θ and -1 respectively, we get the results mentioned on pages 36 and 37.